Girraween High School

2024 Year 12 Mathematics Extension 1
Task 3, July-2024

General Instructions

- Working Time - 90 minutes
- Write using black or blue pen only
- Calculators approved by NESA may be used
- For questions in Section II, Show relevant athematics reasoning and/ or calculations

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Total Marks: 75

Section I - 5 marks (pages 1-3)

- Attempt questions 1-5
- Allow about 10 minutes for this section

Section II - 70 marks (pages 4-8)

- Attempt questions 6-11
- Allow about 80 minutes for this section

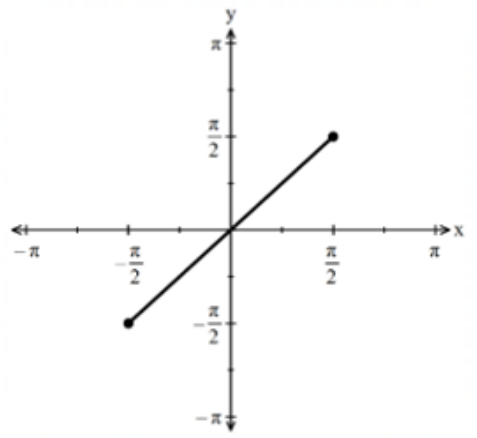
Section I

Multiple Choice Questions

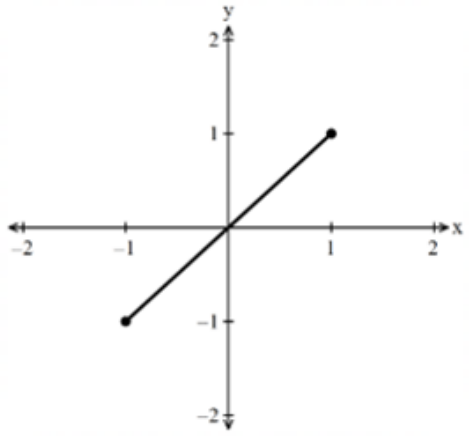
Question 1 (1 mark)

Which of the following sketches represent $y = \cos^{-1}(\cos(x))$?

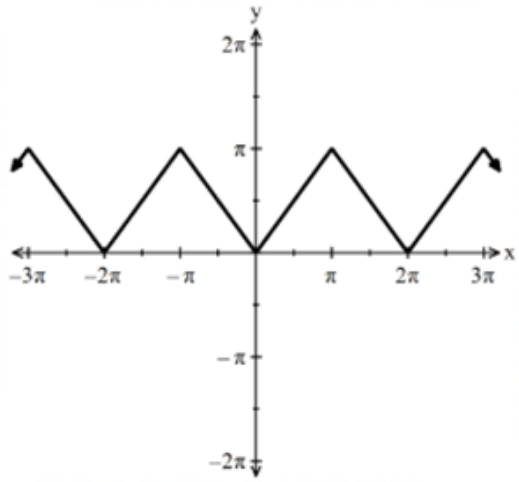
A.



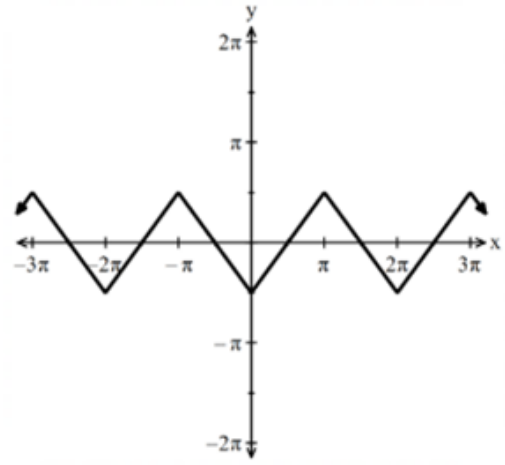
B.



C.



D.



Question 2 (1 mark)

Consider a projectile launched with an initial velocity v at an angle θ to the horizontal. Assume that $g = 9.8ms^{-2}$ and that air resistance is negligeble. Which of the following statements is correct?

A. The acceleration of the projectile decreases during its upward flight.

B. The acceleration of the projectile remains constant during its entire flight.

C. The acceleration of the projectile is greatest during its upward flight.

D. The acceleration of the projectile increases during its downward flight.

Question 3 (1 mark)

What is the value of $\displaystyle\int_{-1}^{1} \frac{x^2}{1+x^2}$

A. $\dfrac{\pi}{2}$

B. 2

C. $2 - \dfrac{\pi}{2}$

D. $2 + \dfrac{\pi}{2}$

Question 4 (1 mark)

The area of a sector in a circle is given by the formula $A = \dfrac{1}{2}r^2\theta$. The radius r is fixed but the angle θ is increasing at a constant rate. The rate of change of A is:

A. zero

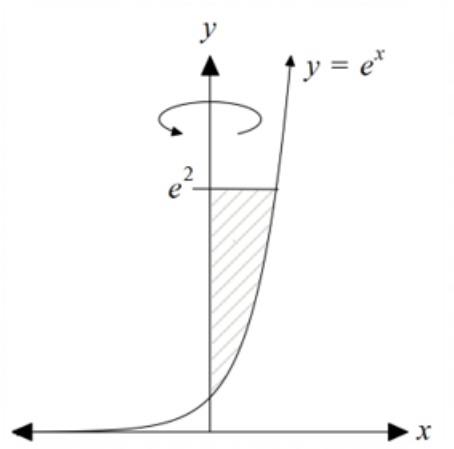
B. decreasing

C. increasing

D. constant

Question 5 (1 mark)

What volume of solid of revolution is formed by rotating the area enclosed by the function $y = e^x$, the line $y = e^2$, and the $y-$ axis about the $y-$ axis.



A. $V = \pi \int_0^2 (ln\ x)^2\ dx$

B. $V = \pi \int_0^2 e^{2x}\ dx$

C. $V = \pi \int_1^{e^2} (ln\ y)^2\ dy$

D. $V = \pi \int_1^{e^2} e^{2y}\ dy$

Mathematics Extension 1

Section II

70 Marks

Attempt Questions 6-11

Allow about 80 minutes for this section

Question 6 (11 marks)

(a) i. Find the domain and range of the function $y = 4\cos^{-1}\left(\frac{x}{3}\right)$ [2]

ii. Sketch the graph of the curve $y = 4\cos^{-1}\left(\frac{x}{3}\right)$ clearly showing the coordinates of the endpoints. [2]

iii. Find the area of the region in the first quadrant bounded by the curve $y = 4\cos^{-1}\left(\frac{x}{3}\right)$ and the coordinate axes. [2]

(b) By writing the expansion of $tan(\alpha - \beta)$, show that [3]
$\tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{4}\right) = \tan^{-1}\left(\frac{2}{9}\right)$

(c) Find $\frac{d}{dx}(x\sin^{-1}x)$ [2]

Question 7 (12 marks)

(a) Solve in the simplest exact form the equation $2\sin^{-1}(2x-1) + \frac{\pi}{2} = 0$. [2]

(b) If $f(x) = tan^{-1}\left(\frac{2}{x}\right)$ find the equation of the tangent to the curve at the point where $x = 2$. [3]

(c) Use the substitution $u = 2x - 1$ to find $\int x\sqrt[3]{2x-1}dx$ [3]

(d) i. Use the substitution $x = \tan^2\theta$ to show that $\int_0^1 \frac{\sqrt{x}}{(1+x)^2}dx = \int_0^{\pi/4} 2\sin^2\theta\, d\theta$ [2]

ii. Hence find in simplest form the value of $\int_0^1 \frac{\sqrt{x}}{(1+x)^2}dx$. [2]

Exam continues on the next page

Question 8 (11 marks)

An astronaut fires a small experimental projectile across a level surface at an angle of 60^o with speed 24m/s. The acceleration due to gravity is 9.8m/s^2

(a) Find the horizontal and vertical components of $V(0)$ [2]

(b) Starting with $\ddot{x} = 0$ and $\ddot{y} = -9.8$. Find $\dot{x}$ and $\dot{y}$. [2]

(c) Hence, find x and y. [2]

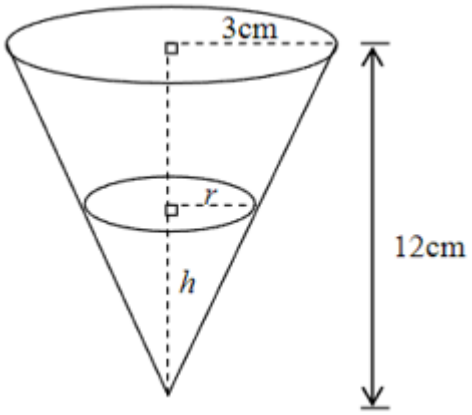
(d) Find the total time of flight. [2]

(e) Find the greatest height reached. [2]

(f) Find the range of the flight. [1]

Question 9 (13 marks)

(a) The diagram shows a conical drinking cup of height 12 cm and radius 3cm.
The cup is being filled with water at the rate of 2cm^3 per second. The height of the water at time t seconds is h cm and the radius of the water's surface is r cm.



i. Show that $r = \frac{1}{4}h$. Give a reason. [2]

ii. Find the rate at which the height is increasing when the height of the water is 8cm. [3]

Question 9 continues on the next page

(b) Heat is applied to a metallic disc for t seconds.
Its area, $A\ cm^2$, increases at a rate given by:

$$\frac{dA}{dt} = t^2 - 2t + 1$$

i. Find the rate at which the area of the metallic disc is increasing at the end of the third second. [1]

ii. Before heat is applied the area of the metallic disc is $10cm^2$. [2]
Heat is then applied to the metallic disc for 3 seconds.
What is the area of this disc at the end of three seconds?

(c) A household iron is cooling in a room of constant temperature 23^oC. At time t minutes its temperature T decreases according to the equation

$\frac{dT}{dt} = -k(T - 23)$ where k is a positive constant.

The initial temperature of the iron is 90^oC and it cools to 70^oC after 10 minutes.

i. Verify that $T = 23 + Ae^{-kt}$ is a solution of this equation where A is a constant. [1]

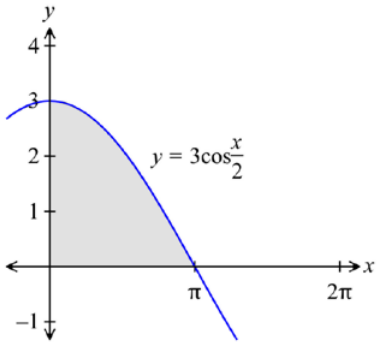
ii. Find the exact values of A and k. [2]

iii. How long will it take for the iron to cool to a temperature of 40^oC? [2]
Give your answer to the nearest minute.

Exam continues on the next page

Question 10 (12 marks)

(a) The region bounded by the graph $y = 3\cos\dfrac{x}{2}$ and the $x-$ axis between $x = 0$ and $x = \pi$ is rotated about the $x-$ axis to form a solid. [3]



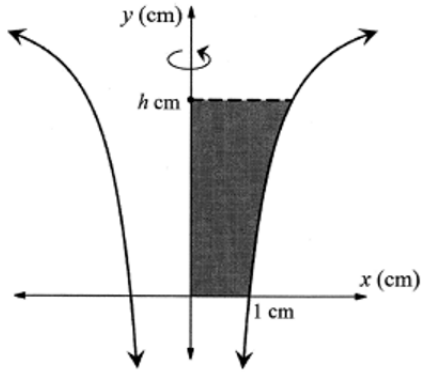
Find the exact volume of the solid.

(b) i. Show that $\displaystyle\int_0^{\pi/4} \cos^2 x\, dx = \frac{\pi + 2}{8}$ [2]

ii. The region under the curve $y = \cos x + \sec x$, above the $x-$ axis, between $x = 0$ and $x = \dfrac{\pi}{4}$, makes a revolution about the $x-$axis. [3]

Show that the volume traced out is $\dfrac{5\pi(\pi + 2)}{8}$ units^3

(c) Consider the shaded region bounded by the curve $y = 9 - \dfrac{9}{x^2}$, the coordinate axes, and the line y=h as shown in the diagram below. [4]



A hollow vase is generated by rotating the section of this curve in the first quadrant about the $y-$axis.

The vase is being filled with water at a constant rate of $50cm^3/min$.

Find the depth h cm of the water at the instant when it is rising at a rate of

$$\frac{50}{3\pi} cm/min.$$

Exam continues on the next page

Question 11 (11 marks)

(a) A particle is projected from a fixed point O on a horizontal plane at an angle of elevation α with speed V metres per second.

After time t, the horizontal and vertical components of its velocity are

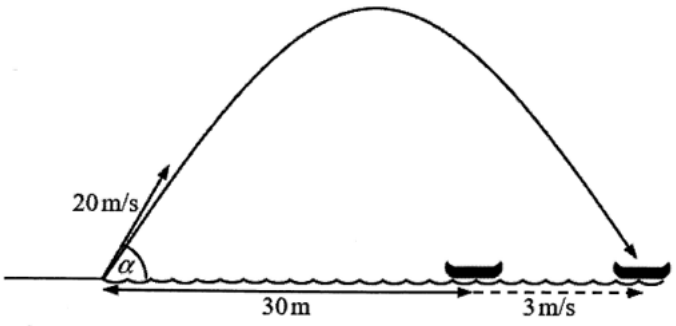
$$\dot{x} = V\cos\alpha \qquad \dot{y} = V\sin\alpha - gt$$

i. Show that the position $P(x, y)$ of the particle at any time as it moves along its path is given by: [2]

$$y = x\tan\alpha - \frac{gx^2}{2V^2}(1+\tan^2\alpha)$$

ii. If the particle is projected from the origin at an angle of 30^o, find the speed required for it to just clear a vertical wall $4m$ high and $12m$ away from the origin. Give your answers correct to 2 decimal places, and take $g = 10ms^{-2}$ [3]

(b) A projectile is thrown at $20\ m/s$ at an angle of inclination α from a lake's edge towards a canoe that is 30 meters away and moving away from shore at $3m/s$ as shown in the diagram below:



The position vector $\underset{\sim}{r}(t)$ of the projectile t seconds after it is thrown is given by

$$\underset{\sim}{r}(t) = \begin{bmatrix} 20t\cos\alpha \\ -5t^2 + 20t\sin\alpha \end{bmatrix} \qquad \text{Do NOT prove this.}$$

i. Show that the projectile lands in the canoe if $20\sin 2\alpha - 6\sin\alpha - 15 = 0$ [2]

ii. Find integers a, b and c such that [1]

$$15x^4 + 92x^3 + 30x^2 - 68x + 15 = (3x^2 + 16x - 5)(ax^2 + bx + c)$$

iii. Use the factorisation in (ii) and the substitution $x = \tan\dfrac{\alpha}{2}$, find solutions for α so that the projectile lands in the canoe. Give your answers correct to the nearest degree. [3]

End of Examination

Final solutions

Mathematics Assessment Solutions
Girraween High School
4/2 Extension 1

Task: 3	Year: 2024

Suggested Solutions

Question 1 (C)

C	B	C	D	C

Question 2 Gravity is constant

Therefore the acceleration of the projectile remains constant during its entire flight.

(B)

Question 3

$$\int_{-1}^{1} \frac{x^2}{1+x^2}\,dx$$

$$= \int_{-1}^{1} \left(1 - \frac{1}{1+x^2}\right) dx = \left(x - \tan^{-1}x\right)_{-1}^{1}$$

$$= 2 - \left(\frac{\pi}{4} + \frac{\pi}{4}\right)$$

$$= 2 - \frac{\pi}{2}$$ (C)

Marker's Feedback

Suggested Solutions

Question (4)

r is constant

$$A = \frac{1}{2} r^2 \theta$$

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$, which is constant (D)

(5)

As the radius of the circular cross-sections are given by the distance between the y-axis and the function, we need to solve for x (find inv)

$$x = \ln y$$

$$V = \pi \int_a^b (f(y))^2 \, dy = \pi \int_1^{e^2} (\ln y)^2 \, dy$$ (C)

Marker's Feedback

Suggested Solutions

Question

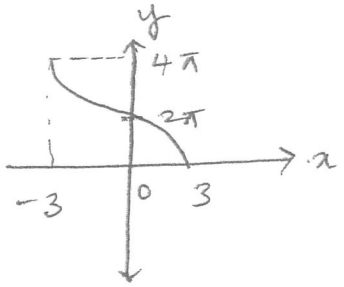
6a, i, $y = 4\cos^{-1}\left(\frac{x}{3}\right)$

$D: \{-3 \le x \le 3\}$

$R: \{0 \le y \le 4\pi\}$

Mostly well done

ii,



Done well

$y = 4\cos^{-1}\left(\frac{x}{3}\right)$

$\frac{y}{4} = \cos^{-1}\left(\frac{x}{3}\right)$

$x = 3\cos\frac{y}{4}$

iii, $A = \int_0^{2\pi} 3\cos\left(\frac{y}{4}\right) dy$

$= \left[12\sin\frac{y}{4}\right]_0^{2\pi}$

$= 12\left(\sin\frac{\pi}{2} - \sin 0\right)$

$= 12$ sq units

Some students did not know how to begin answering this question

Marker's Feedback

Suggested Solutions

Question 6b

let $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$ and $\beta = \tan^{-1}\left(\frac{1}{4}\right)$

$\therefore \tan\alpha = \frac{1}{2}$ and $\tan\beta = \frac{1}{4}$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\left(\frac{1}{2} - \frac{1}{4}\right)}{1 + \left(\frac{1}{2} \times \frac{1}{4}\right)}$$

$$\therefore \tan(\alpha-\beta) = \frac{2}{9}$$

$$\alpha - \beta = \tan^{-1}\left(\frac{2}{9}\right)$$

$$\therefore \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{4}\right) = \tan^{-1}\left(\frac{2}{9}\right)$$

Marker's Feedback

With the given hint, most students figured out how to answer this questi

Suggested Solutions

Question

6C

$\frac{d}{dx}(x \sin^{-1} x)$

Using product rule: $u'v + uv'$

$= \sin^{-1} x + \frac{x}{\sqrt{1-x^2}}$

$\frac{d}{dx}(x \sin^{-1} x) = \frac{x}{\sqrt{1-x^2}} + \sin^{-1} x$

let $u = x$

$u' = 1$

$v = \sin^{-1} x$

$v' = \frac{1}{\sqrt{1-x^2}}$

Very well done.

Marker's Feedback

Suggested Solutions

Question

Q7,9,

$$2\sin^{-1}(2x-1) + \frac{\pi}{2} = 0$$

$$\sin^{-1}(2x-1) = -\frac{\pi}{4}$$

$$(2x-1) = \frac{-1}{\sqrt{2}}$$

$$2x = 1 - \frac{\sqrt{2}}{2}$$

$$x = \frac{2-\sqrt{2}}{4}$$

Generally well done.

Marker's Feedback

Suggested Solutions

Question

~~Q7d~~ (7b)

$f(x) = \tan^{-1}\left(\frac{2}{x}\right)$

$f'(x) = \dfrac{\frac{d}{dx}(2x^{-1})}{1+\left(\frac{2}{x}\right)^2}$

$= \dfrac{\left(\frac{-2}{x^2}\right)}{1+\frac{4}{x^2}}$

Differentiation was poorly done.

$\therefore f'(x) = \dfrac{-2}{x^2+4}$ and $f'(2) = \dfrac{-2}{2^2+4} = -\frac{1}{4}$

Some did not evaluate $f'(2)$

when $x=2$, $\tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

Marker's Feedback

Suggested Solutions

Question 7b continued

∴ equation of tangent with $m = \frac{-1}{4}$, passing through $(2, \frac{\pi}{4})$ is

$$y - \frac{\pi}{4} = \frac{-1}{4}(x-2)$$

This part well done.

$$4y - \pi = -x + 2$$

∴ The equation of the tangent is

$$x + 4y - \pi - 2 = 0$$

Marker's Feedback

Suggested Solutions

Question 7C

$$\int x \sqrt[3]{2x-1}\, dx$$

let $u = 2x-1$

$x = \frac{1}{2}(u+1)$

$$\frac{1}{2}\int x \sqrt[3]{2x-1}\, 2\,dx$$

Also $u = 2x-1$

$du = 2dx$

$\frac{1}{2}du = dx$

many wrote $u^{1/2}$

$$\frac{1}{2}\int \frac{1}{2}(u+1)\cdot u^{1/3}\, du$$

$$\frac{1}{4}\int (u^{4/3} + u^{1/3})\, du$$

$$\frac{1}{4}\left[\frac{3u^{7/3}}{7} + \frac{3u^{4/3}}{4}\right]$$

$$\frac{3u^{1/3}}{28}(u^7 + u^4)$$

$$\frac{3\sqrt[3]{2x-1}}{28}((2x-1)^7 + (2x-1)^4)$$

Many students did not write in terms of x.

Marker's Feedback

This question was omitted from the total marks.

Suggested Solutions

Question

7d, i, $x = \tan^2\theta$, $0 \le \theta < \frac{\pi}{2}$

$dx = 2\tan\theta \sec^2\theta \, d\theta$

changing limits

$x = 0 \Rightarrow \theta = 0$

$x = 1 \Rightarrow \theta = \frac{\pi}{4}$

$$\int_0^1 \frac{\sqrt{x}}{(1+x^2)^2} dx = \int_0^{\pi/4} \frac{\tan\theta}{(\sec^2\theta)^2} \cdot 2\tan\theta \sec^2\theta \, d\theta$$

$$= \int_0^{\pi/4} \frac{2\tan^2\theta}{\sec^2\theta} \, d\theta$$

$$= \int_0^{\pi/4} 2\sin^2\theta \, d\theta = \ldots$$

Marker's Feedback

Suggested Solutions

Question

7d(ii),

$$\int_0^1 \frac{\sqrt{x}}{(1+x)^2}\,dx = \int_0^{\pi/4} 2\sin^2\theta\,d\theta$$

$$= \int_0^{\pi/4} (1-\cos 2\theta)\,d\theta$$

$$= \left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi/4}$$

$$= \frac{\pi}{4} - \frac{1}{2}(1-0)$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

well done

Marker's Feedback

Suggested Solutions

Question 8

Horizontal component of $\underset{\sim}{v}(0)$

$24(\cos 60°)$

$= 24\left(\frac{1}{2}\right)$

$= 12$

Vertical component of $\underset{\sim}{v}(0)$

$24(\sin 60°)$

$24\left(\frac{\sqrt{3}}{2}\right)$

$= 12\sqrt{3}$

(b) Finding $\underset{\sim}{v}(t)$

Horizontal:

From (a)

$= 12$

Vertical

$\underset{\sim}{v}(t) = \int -9.8\, dt$

$= -9.8t + c_1$

From (a) at $t = 0$

$\underset{\sim}{v}(0) = 12\sqrt{3}$

$12\sqrt{3} = -9.8(0) + c_1$

$\therefore \underset{\sim}{v}(t) = -9.8t + 12\sqrt{3}$

Marker's Feedback

students were bit confused because the answers for (a) comes in the working out for (b)

Suggested Solutions

Question 8b continued.

Horizontal

$= \int 12\,dt$

$= 12t$

when $t=0$; $x(0)=0$

$=12t$

Vertical

$= \int (12\sqrt{3} - 9.8t)\,dt$

$= 12\sqrt{3}t - 4.9t^2 + c_2$

$\therefore \quad c_2 = 0$

$= 12\sqrt{3}t - 4.9t^2$

(8c) Total time of flight

Sub $12\sqrt{3}t - 4.9t^2 = 0$

$t(12\sqrt{3} - 4.9t) = 0$; $t=0$; $12\sqrt{3} - 4.9t = 0$

$t=0$ and $t = \dfrac{12\sqrt{3}}{4.9}$ sec

$\therefore$ The total time of flight is $\dfrac{12\sqrt{3}}{4.9}$ seconds

Marker's Feedback

* Many did not show the correct working out, when deriving the equations.
* Some got only one answers when solving $12\sqrt{3}t - 4.9t^2 = 0$. They lost 1 mark.

Suggested Solutions

Question 8d

Greatest height is reached at $t = \left(\frac{12\sqrt{3}}{4 \cdot 9}\right) \div 2$

i.e $t = \frac{12\sqrt{3}}{9 \cdot 8}$

Sub $t = \frac{12\sqrt{3}}{9 \cdot 8}$ in $12\sqrt{3}\,t - 4 \cdot 9\,t^2$ (vertical component of $r(t)$)

$$= 12\sqrt{3}\left(\frac{12\sqrt{3}}{9 \cdot 8}\right) - 4 \cdot 9\left(\frac{12\sqrt{3}}{9 \cdot 8}\right)^2$$

$$= \left(\frac{12\sqrt{3}}{9 \cdot 8}\right)^2 \left(1 - \frac{4 \cdot 9}{9 \cdot 8}\right)$$

$$= \left(\frac{12\sqrt{3}}{9 \cdot 8}\right)^2 \times \frac{1}{2}$$

$$= 22.04 \text{ m}$$

Marker's Feedback

done well

Suggested Solutions

Question 8e

Range of flight

Sub $t = \frac{12\sqrt{3}}{4 \cdot 9}$ in $12t$ (Horizontal component of $x(t)$)

$\therefore$ Range $= 12 \times \frac{12\sqrt{3}}{4 \cdot 9}$

$= 50 \cdot 90\text{ m}$ (2 d.p)

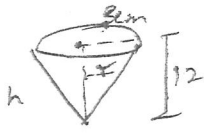
Marker's Feedback

done well

Suggested Solutions

Question

(9a) $V = \frac{1}{3}\pi r^2 h$ and $r = \frac{h}{4}$ (Using similar Δ^{les})



$\frac{r}{8} = \frac{h}{12}$

$r = \frac{h}{14}$

Poorly set out, students should mention similar triangle as a reason and ratio of matching sides.

$\therefore V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h$ $\quad \left(\frac{h}{4}\right)^2 \neq \frac{h^2}{4}$

$V = \frac{\pi h^3}{48}$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$

Lot of students did $\frac{dV}{dh}$ without changing r in terms of h.

$\frac{dh}{dt} = \frac{16}{\pi h^2} \times 2$

$\frac{dh}{dt} = \frac{32}{\pi h^2}$ since $h = 8$

$\frac{dh}{dt} = \frac{1}{2\pi}$ cm/s

$\frac{dV}{dh} = \frac{3\pi h^2}{48} = \frac{\pi h^2}{16}$

Marker's Feedback

Suggested Solutions

Question

Qb

Mostly done well but some students put $t=4$

(i) At $t=3$

$$\frac{dA}{dt} = (3)^2 - 2(3) + 1$$

$$= 4$$

The rate at which disc is increasing is $4\,cm^2/s$

(ii) $$A = \int (t^2 - 2t + 1)\,dt$$

Mostly done well.

$$= \frac{t^3}{3} - t^2 + t + C$$

when $t=0$, $A = 10 \Rightarrow C = 10$

$$\therefore A = \frac{1}{3}t^3 - t^2 + t + 10$$

At $t=3$, $A = \frac{1}{3}(3)^3 - (3)^2 + 3 + 10$

$$= 13\,cm^2$$

Marker's Feedback

Suggested Solutions

Question

(9C) i) $T = 23 + Ae^{-kT}$

$Ae^{-kt} = T - 23$

$\frac{dT}{dt} = -kAe^{-kt}$

$= -k(T-23)$

Some students did not set out the working properly.

ii) $t = 0$ and $T = 90$

$\therefore 90 = 23 + Ae^{-k(0)}$

$\therefore A = 67$

Mostly done well.

$t = 10$ and $T = 70$

$\therefore 70 = 23 + 67e^{-k(10)}$

$k = -\frac{1}{10}\ln\left(\frac{47}{67}\right)$

Marker's Feedback

Suggested Solutions

Question

9c iii, $T = 23 + 67e^{-kt}$ where $k = \frac{-1}{10}\ln\left(\frac{47}{67}\right)$

$40 = 23 + 67e^{-kt}$

$17 = 67e^{-kt}$

$\left(\frac{17}{67}\right) = e^{-kt}$

$t = \frac{\ln\left(\frac{17}{67}\right)}{-k}$

Done well.

$t = 38.68$ min

$t = 39$ mins

time left as degrees and minutes wrong notation.

380ff!

Marker's Feedback

Suggested Solutions

Question 10a

$$V = \pi \int_a^b y^2 \, dx$$

$$= \pi \int_0^{\pi} \left(3 \cos \frac{x}{2}\right)^2 dx$$

$$= \pi \int_0^{\pi} 9 \cos^2 \frac{x}{2} \, dx$$

$$= \frac{9\pi}{2} \int_0^{\pi} 2 \cos^2 \frac{x}{2} \, dx$$

$$= \frac{9\pi}{2} \int_0^{\pi} (1 + \cos x) \, dx$$

$$= \frac{9\pi}{2} \left[x + \sin x \right]_0^{\pi}$$

Marker's Feedback

Generally OK.

Suggested Solutions

Question

10a Continued

$$= \frac{9\pi}{2}\left[(\pi + \sin\pi) - (0 + \sin 0)\right]$$

$$= \frac{9\pi^2}{2} \text{ cubic units}$$

Marker's Feedback

Generally OK.

Suggested Solutions

Question

(10b) i, $\cos 2x = 2\cos^2 x - 1$

$$\cos^2 x = \frac{1}{2}(\cos 2x + 1)$$

$$\int_0^{\pi/4} \cos^2 x \, dx = \frac{1}{2}\int_0^{\pi/4} (\cos 2x + 1)\, dx$$

$$= \frac{1}{2}\left[\frac{1}{2}\sin 2x + x\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left[\frac{1}{2}\sin\frac{\pi}{2} + \frac{\pi}{4}\right]$$

$$= \frac{1}{2}\left[\frac{1}{2} + \frac{\pi}{4}\right]$$

$$= \frac{\pi+2}{8}$$

Marker's Feedback

A few students forgot $\cos^2 x = \frac{1}{2}\cos 2x + \frac{1}{2}$.

Suggested Solutions

Question

10b)ii), $y = \cos x + \sec x$

$$y^2 = \cos^2 x + \sec^2 x + 2$$

$$V = \pi \int_0^{\pi/4} y^2 \, dx$$

$$= \pi \int_0^{\pi/4} \cos^2 x \, dx + \pi \int_0^{\pi/4} (\sec^2 x + 2) \, dx$$

$$= \pi \left[\frac{\pi + 2}{8} \right] + \pi \left[\tan x + 2x \right]_0^{\pi/4}$$

$$= \pi \left[\frac{\pi + 2}{8} \right] + \pi \left[\tan \frac{\pi}{4} + \frac{\pi}{2} \right]$$

$$= \pi \left[\frac{\pi + 2}{8} + 1 + \frac{\pi}{2} \right]$$

$$= \pi \left[\frac{\pi + 2 + 8 + 4\pi}{8} \right]$$

$$= \frac{5\pi(\pi + 2)}{8}$$

Marker's Feedback

At least 10 students think $(x+y)^2 = x^2 + y^2$???
i.e. $(\cos x + \sec x)^2 = \cos^2 x + \sec^2 x$.
→ This is a VERY bad mistake.
Should be $(\cos x + \sec x)^2 = \cos^2 x + 2\cos x \sec x + \sec^2 x$
$= \cos^2 x + \sec^2 x + 2$.

A lot of students didn't use their answer to (i).

Suggested Solutions

Question

(10C) $y = 9 - \frac{9}{x^2}$

Making x^2 the subject of the equation gives

$$\frac{9}{x^2} = 9 - y$$

$$x^2 = \frac{9}{9-y}$$

$$\therefore V = \pi \int_0^h x^2 \, dy$$

Alternative to taking the shortcut ↗

$$V = \pi \int_0^h \frac{9}{9-y} \, dy$$

$$V = -9\pi \left[\ln(9-y) \right]_0^h$$

$$V = -9\pi \ln\left(\frac{9-h}{9}\right)$$

$$\frac{dV}{dh} = -9\pi \times -\frac{1}{9} \times \frac{9}{9-h}$$

$$\frac{dV}{dh} = \frac{9\pi}{9-h}$$

Differentiating both sides with respect to h

$$\frac{dV}{dh} = \pi \left(\frac{9}{9-h}\right)'$$

Since $\frac{dV}{dt} = 50$ and $\frac{dh}{dt} = \frac{50}{3\pi}$, we have

$$\frac{dV}{dh} = \frac{dV}{dt} \div \frac{dh}{dt} = 50 \div \frac{50}{3\pi} = 3\pi$$

Marker's Feedback

Very poorly done. → A few students didn't realise that they already had x^2 and didn't need to SQUARE it.
→ The vast majority of students had no idea how to handle the chain rule with rates of change.

Suggested Solutions

Question 10c, continued

Accordingly,

$$3\pi = \pi\left(\frac{9}{9-h}\right)$$

$$3(9-h) = 9$$

$$h = 6$$

The depth of the water at the instant when it is rising at the given rate will be 6 cm.

Marker's Feedback

Suggested Solutions

Question || a ||

$\ddot{x} = V\cos\alpha \qquad \ddot{y} = V\sin\alpha - g$

Integrating

$\dot{x} = Vt\cos\alpha + C_1 \qquad y = Vt\sin\alpha - \frac{1}{2}gt^2 + C_2$

Initially $x = 0$ and $y = 0$. Hence $C_1 = C_2 = 0$

$\therefore x = Vt\cos\alpha \; ; \; y = Vt\sin\alpha - \frac{1}{2}gt^2 \quad -(2)$

$t = \frac{x}{V\cos\alpha} \quad -(1)$

Sub (1) into (2)

$y = V\left(\frac{x}{V\cos\alpha}\right)\sin\alpha - \frac{1}{2}g\left(\frac{x}{V\cos\alpha}\right)^2$

$y = x\tan\alpha - \frac{gx^2}{2V^2}\sec^2\alpha$

$y = x\tan\alpha - \frac{gx^2}{2V^2}(1+\tan^2\alpha)$; as required.

Marker's Feedback

Most students did this well

Suggested Solutions

Question , 11a, ii,

Sub $\alpha = 30^\circ$, $x = 12$ & $y = 4$ in

$$y = x\tan\alpha - \frac{gx^2}{2V^2}(1+\tan^2\alpha)$$

$$4 = \frac{12}{\sqrt{3}} - \frac{120\left(1+\frac{1}{3}\right)}{V^2}$$

$$4 = \frac{12}{\sqrt{3}} - \frac{960}{V^2}$$

$$\frac{960}{V^2} = \frac{12}{\sqrt{3}} - 4$$

$$V^2 = \frac{960}{\frac{12}{\sqrt{3}} - 4}$$

$$V = \sqrt{\frac{960}{\frac{12}{\sqrt{3}} - 4}} \approx 18.10 \text{ m/s}$$

Marker's Feedback

Mostly done well, but some students did some Calculation mistakes

Suggested Solutions

Question

(11b) i, The projectile lands when the vertical position is zero, that is

$$-5t^2 + 20t\sin\alpha = 0$$

$$-5t(t - 4\sin\alpha) = 0$$

$$t = 0 \quad \text{or} \quad 4\sin\alpha$$

Now, the horizontal position x m of the canoe is given by $x = 30 + 3t$ and the horizontal position x m of the projectile will is given by $x = 20t\cos\alpha$,

so the projectile will land in the canoe if these expressions are equal when $t = 4\sin\alpha$,

so

$$20(4\sin\alpha)\cos\alpha = 30 + 3(4\sin\alpha)$$

$$20(2\sin\alpha\cos\alpha) = 15 + 6\sin\alpha$$

$$20\sin 2\alpha - 6\sin\alpha - 15 = 0 \quad \text{as required}$$

Marker's Feedback

Some students have shown progress in finding $t = 4\sin\alpha$ and only few students could successfully do the second part of the question to get to $20\sin 2\alpha - 6\sin\alpha - 15 = 0$

Suggested Solutions

Question

11b (ii),

$15x^4+92x^3+30x^2-6x+15 = (3x^2+16x-5)(ax^2+bx+c)$

$$
\begin{array}{r}
5x^2+4x-3 \\
3x^2+16x-5 \overline{)\,15x^4+92x^3+30x^2-68x+15} \\
15x^4+80x^3-25x^2 \\
\hline
12x^3+55x^2-68x \\
12x^3+64x^2-20x \\
\hline
-9x^2-48x+15 \\
-9x^2-48x+15 \\
\hline
0
\end{array}
$$

Hence, $15x^4+92x^3+30x^2-68x+15 = (3x^2+16x-5)(5x^2+4x-3)$

$\therefore a=5;\ b=4;\ c=-3$

Marker's Feedback

Mostly done well!

Suggested Solutions

Question

11b(iii)

Making t-substitutions, $x = \tan\frac{\alpha}{2}$ gives

$$\sin\alpha = \frac{2x}{1+x^2} \quad \text{and} \quad \cos\alpha = \frac{1-x^2}{1+x^2}$$

Substituting into the equation

$20 \times 2\sin\alpha\cos\alpha - 6\sin\alpha - 15 = 0$ from (i)

$$20 \times 2 \times \frac{2x}{1+x^2} \times \frac{1-x^2}{1+x^2} - 6 \times \frac{2}{1+x^2} - 15 = 0$$

$$80(x - x^3) - 12(x + x^3) - 15(1+x^2)^2 = 0$$

$$80x - 80x^3 - 12x - 12x^3 - 15 - 30x^2 - 15x^4 = 0$$

$$15x^4 + 92x^3 + 30x^2 - 68x + 15 = 0$$

$$(3x^2 + 16x - 5)(5x^2 + 4x - 3) = 0 \quad \text{from (ii)}$$

Marker's Feedback

Students have done this in a couple of ways but only very few number of students were successful in getting the right answers.

Suggested Solutions

Question 11 b(ii), continued

$3x^2 + 16x - 5 = 0$

$\therefore x = \dfrac{-16 \pm \sqrt{16^2 - 4 \times 3 \times (-5)}}{6}$

$5x^2 + 4x - 3 = 0$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4 \times 5 \times (-3)}}{10}$

But since α is acute and $x = \tan\frac{\alpha}{2}$, we can eliminate the negative values of x.

$\alpha = 2\tan^{-1}\left(\dfrac{-8 + \sqrt{79}}{3}\right)$ or $2\tan^{-1}\left(\dfrac{-2 + \sqrt{19}}{5}\right)$

$\simeq 33^\circ$ or 57°

Marker's Feedback